Seismic Performance-Based Design: Using Neural Networks for Reliability Computation and Optimization

Ricardo O. Foschi
Department of Civil Engineering
University of British Columbia

Summary:

In this talk I want to discuss a general definition for what is called "Performance-Based Design", and will try to identify the main components of such a task. I will point out the close relationship between "Performance-Based Design" and the consideration of uncertainties.

I will do so in the context of earthquake engineering, and illustrate the procedure using the design of a reinforced concrete frame with several stories.

Finally, I will point out what further research is needed in this area.

What Is PERFORMANCE - BASED DESIGN ?

It is an **Optimization**, to find a set of optimum design parameters (**d**), satisfying minimum prescribed reliability levels for each desired performance requirement, and possibly minimizing an objective function.

The objective function could be, for example, the total cost $F(\mathbf{d})$:

Total cost:

$$\boxed{F(\mathbf{d}) = C_0(\mathbf{d}) + C_1(\mathbf{d})}$$

C_0 : The initial construction cost, including materials (e.g., concrete and steel), forming and labor.

C_1 : The future repair or replacement cost, at present values, which depends on the level of damage during a future event within the life of the structure, and the discount rate available to accumulate repair funds.

Repair cost at present values

We need to quantify the level of damage done by the earthquake. We could use a *global damage index DIG (0 < DIG < 1)* as an indicator of structural distress, and have this damage *DIG* directly related to calculated deformations or displacements. If $C_f(DIG)$ is the repair cost in terms of *DIG* *for an earthquake occurring at an unkown future time t*, then the present cost $C_{f\,0}(DIG)$ is, r being the discount or interest rate :

$$C_{f\,0}(DIG) = C_f\,(DIG)\ \exp(-r\,t)$$

When the arrival time of earthquakes is simulated as a Poisson process with rate ν, the expected present cost conditional on the damage *DIG* can be shown to be :

$$C_1\,|_{DIG} = \int_0^\infty C_{f\,0}(DIG)\ \ f\ (t)\ dt = C_f\,(DIG)\ \frac{\nu}{r+\nu}$$

In seismic engineering, the event is an earthquake.

Since the earthquake may trigger nonlinear structural responses, we need

1) A nonlinear dynamic analysis of the structure under earthquake excitation, which will give the values of displacements, stresses, forces, etc., during the earthquake, as well as their maximum values. The analysis would automatically calculate the hysteretic response.

2) A relationship between the calculated quantities and a measure of damage (damage index *DIG*). For this we need a damage model (for example, Park and Ang's damage index or something similar) . The index must relate not only to the maximum response but also to the deformation history (hysteresis).

Furthermore, there is **uncertainty** in the **structural properties** and, certainly, **in the ground motions**.

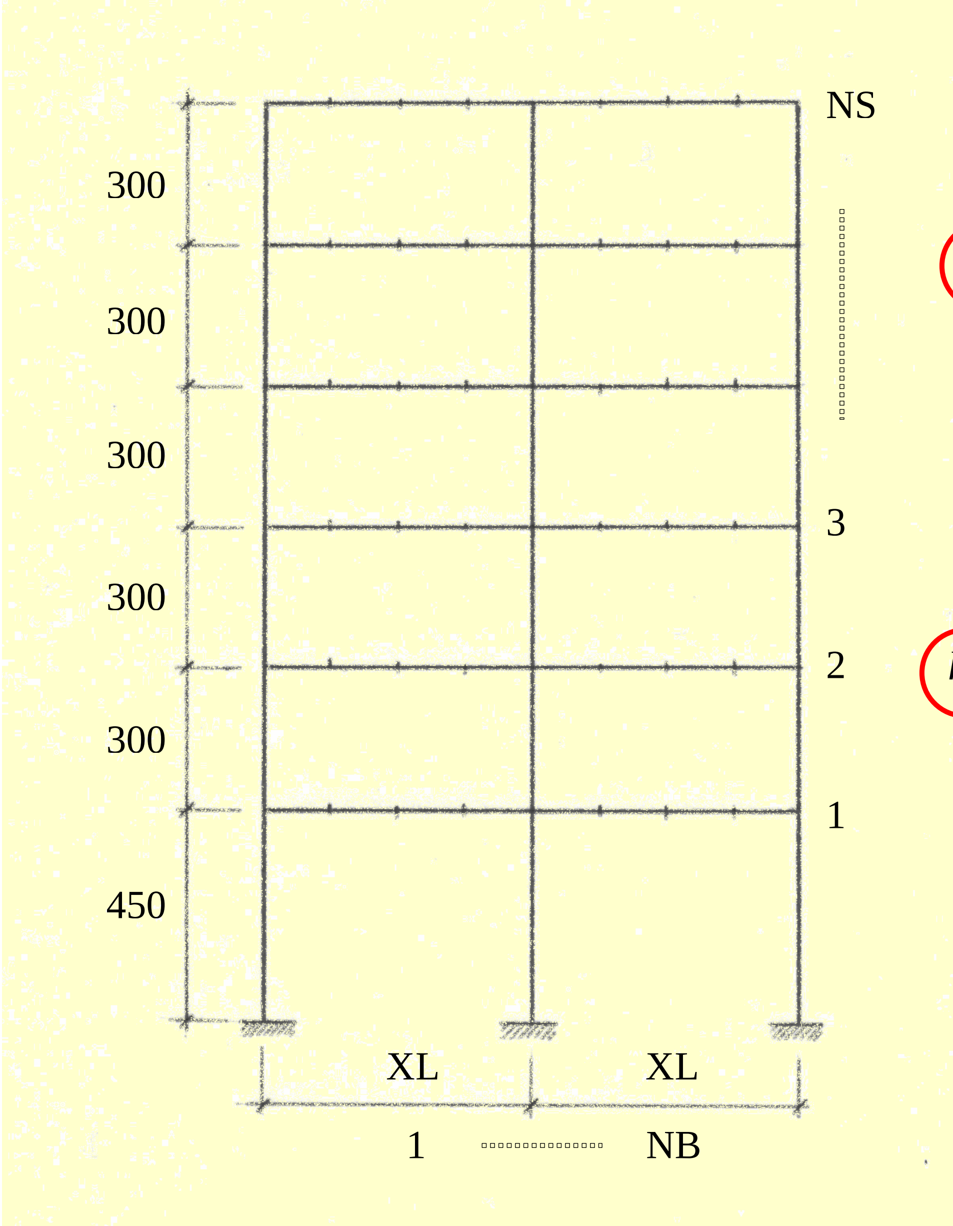
Therefore, the calculated extreme displacements and/or actions, and estimated damage, will also be random.

This randomness leads to uncertainty in meeting the required performance levels.

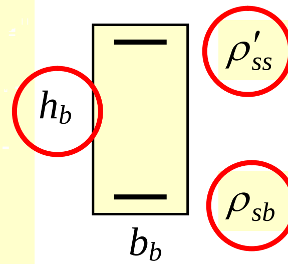
It is required to define the performance levels and the minimum target reliabilities associated with each of them.

Thus, we need a **reliability analysis** to calculate the achieved reliabilities for a given set of design variables, in order to check that they satisfy the minimum targets.

To illustrate, consider an application and the desired design parameters (d), shown in circles:



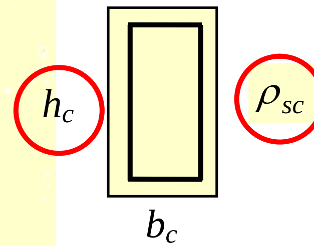
Beam sections



$\rho'_{ss} = A'_s/(b_b h_b)$

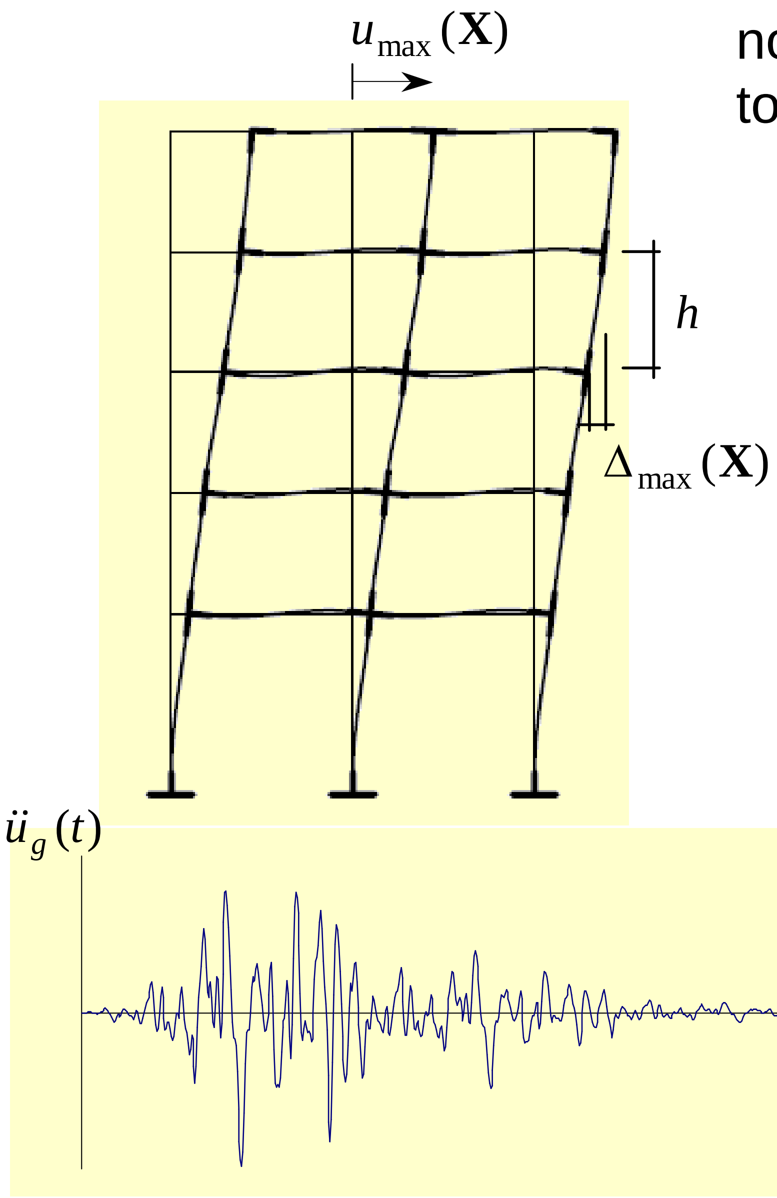
$\rho_{sb} = A_s/(b_b h_b)$

Column sections



$\rho_{sc} = A_{sc}/(b_c h_c)$

NONLINEAR DYNAMIC ANALYSIS AND RESPONSES



If **X** is the set of random variables, nonlinear dynamic analysis can be used to obtain the following responses:

Maximum lateral displacement:

$$u_{\max}(\mathbf{X})$$

Maximum inter-story drift:

$$DSIM(\mathbf{X}) = \frac{\Delta_{\max}(\mathbf{X})}{h}$$

Maximum local Park and Ang damage index:

$$DILOM(\mathbf{X})$$

Global damage index:

$$DIG(\mathbf{X})$$

RELIABILITY ESTIMATION FOR DIFFERENT PERFORMANCE LEVELS

General form of the Limit State functions $G(\mathbf{X}) = RLIM - R(\mathbf{X})$

a) Performance level : Operational

- **Elastic behavior** $G_{11}(\mathbf{X}) = (\bar{u}_y, 0.10) - u_{\max}(\mathbf{X})$
- **Inter-story drift** $G_{12}(\mathbf{X}) = (0.005, 0.10) - DSIM(\mathbf{X})$

b) Performance level : Life safety

- **Inter-story drift** $G_{21}(\mathbf{X}) = (0.015, 0.10) - DSIM(\mathbf{X})$
- **Maximum local damage index** $G_{22}(\mathbf{X}) = (0.60, 0.10) - DILOM(\mathbf{X})$
- **Global damage index** $G_{23}(\mathbf{X}) = (0.40, 0.10) - DIG(\mathbf{X})$

c) Performance level : Near collapse

- **Inter-story drift** $G_{31}(\mathbf{X}) = (0.025, 0.10) - DSIM(\mathbf{X})$
- **Maximum local damage index** $G_{32}(\mathbf{X}) = (1.00, 0.10) - DILOM(\mathbf{X})$
- **Global damage index** $G_{33}(\mathbf{X}) = (0.80, 0.10) - DIG(\mathbf{X})$

Example of Statistics for the Random Variables **X**

Variable	$\bar{X}$	σ_X	Type	Variable	$\bar{X}$	σ_X	Type
$X(1) = NS$	6	0	Normal	$X(13) = f_r / f'_{c0}$	0.10	0.01	Normal
$X(2) = NB$	3	0	Normal	a_G	$a_G = X(14)\,[1.0 + X(15)]$		
$X(3) = XL$	400 *cm*	20.0 *cm*	Normal	$X(14) = \bar{a}_G$	94 cm/s^2	130 cm/s^2	Lognormal
$X(4) = m$	$2.5\text{x}10^{-4}$	$2.5\text{x}10^{-5}$	Normal	$X(15) = \sigma_{\bar{a}_G}$	0	0.25	Normal
$X(5) = f'_c$	30 *MPa*	3 *MPa*	Lognormal	$X(16) = f_g$	2.50 *Hz*	0.375 *Hz*	Normal
$X(6) = b_b$	20 *cm*	1 *cm*	Normal	$X(17) = X_{N1}$	0	1	Normal
$X(7) = h_b$	? *cm*	0.05 $\bar{X}$	Normal	$X(18) = X_{N2}$	0	1	Normal
$X(8) = b_c$	30 *cm*	1.5 *cm*	Normal	$X(19) = R_{N1}$	0	1	Normal
$X(9) = h_c$	? *cm*	0.05 $\bar{X}$	Normal	$X(20) = R_{N2}$ (*)	0	1	Uniform
$X(10) = \rho_{sb}$	?	0.10 $\bar{X}$	Lognormal	$X(21) = R_{N3}$	0	1	Normal
$X(11) = \rho_{ss}$	?	0.10 $\bar{X}$	Lognormal	$X(22) = R_{N4}$ (*)	0	1	Uniform
$X(12) = \rho_{sc}$	?	0.10 $\bar{X}$	Lognormal	(*) Bounds for Uniform distribution			

Design parameters d are the mean values of ***X*(7)**, ***X*(9)**, ***X*(10)**, ***X*(11)** and ***X*(12)**

Probability of failure or non-performance:

$$Pf = \mathrm{Prob}[\, G(\mathbf{X}) \leq 0 \,]$$

This calculation can be done by different methods:

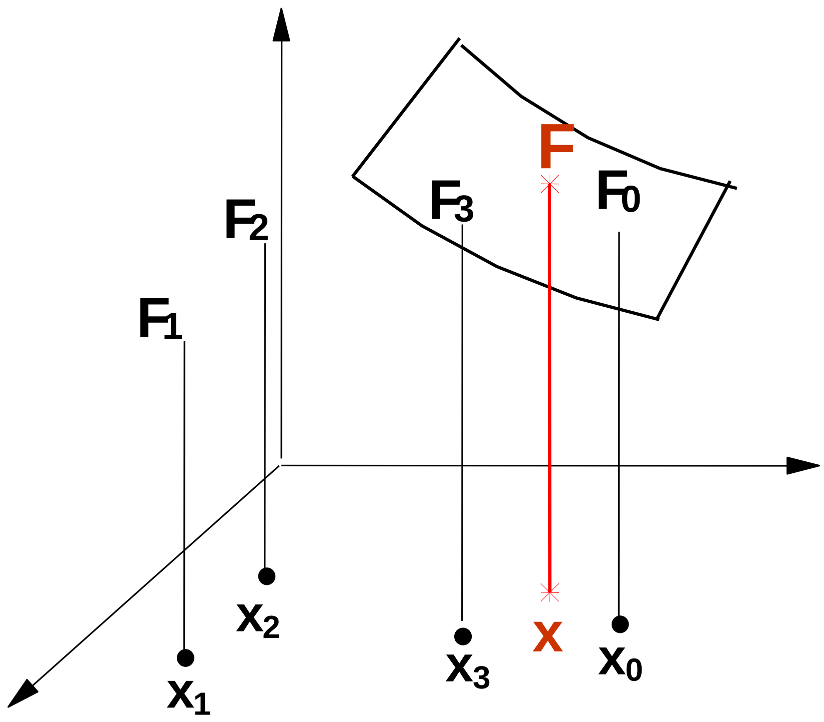
1) Simulation: randomly select the vector X and check whether G < 0 or not, compile all values leading to G < 0.

2) FORM: an approximate, efficient method which calculates a reliability index β and, from it, the probability of failure.

Both methods require the evaluation of G given X.

This could be done every time by repeating the dynamic analysis.

Or the dynamic analysis could be run ahead of time to develop information on the relationship between X and G (a **"response surface"**).



A **response surface** permits interpolation of a response for a variable combination **x**, after it has been trained to known data (x_i , F_i).

Implementing the dynamic analysis every time the G are needed may be computationally demanding.

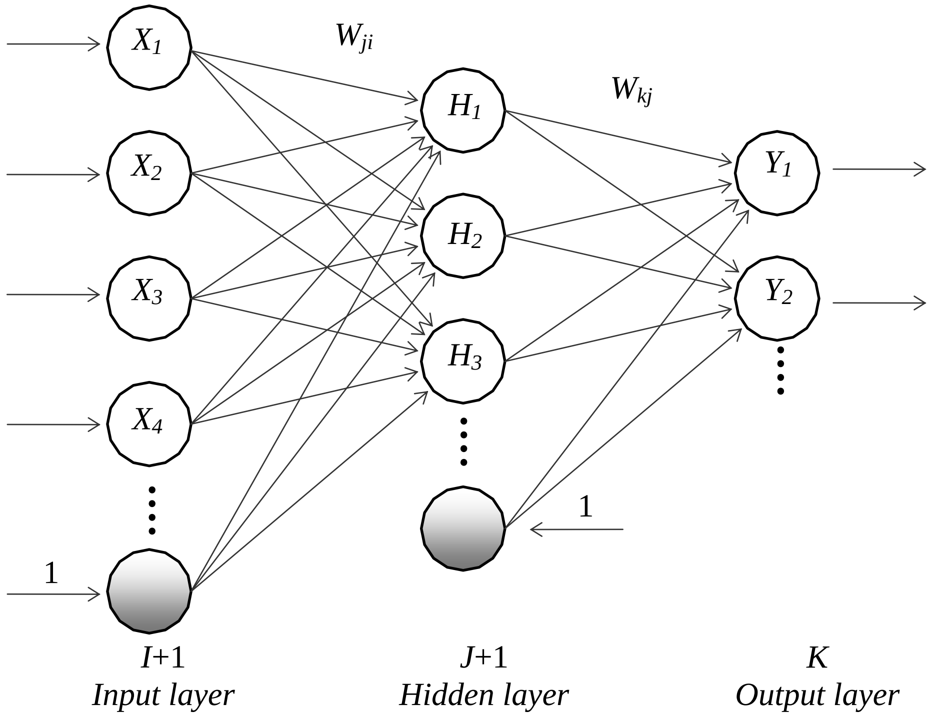
A response surface acts then as a substitute for the dynamic analysis.

When developed, using the response surface in simulations or FORM becomes very efficient and fast.

Building the database for the response surface is also computationally demanding but, when built, it functions as a library of responses that can be very efficiently used for reliability estimation and design optimization.

When a response surface is built, running simulations becomes very feasible. One could also use FORM, but this method has the disadvantage that its results depend on the nonlinearity of the G function. Simulations do not have this disadvantage.

A NEURAL NETWORK: A type of response surface



Multi-layer neural network
(1 hidden layer)

$$\boxed{R(\mathbf{X}) \cong F(\mathbf{X}) = h\left(\sum_{k=0}^{J} W_{kj}\, h\left(\sum_{i=0}^{N} W_{ji}\, X_i\right)\right)}$$

Training: find the optimum parameters W to best represent the output data

Approach to obtaining the statistics for the seismic responses $\boldsymbol{R_i}$

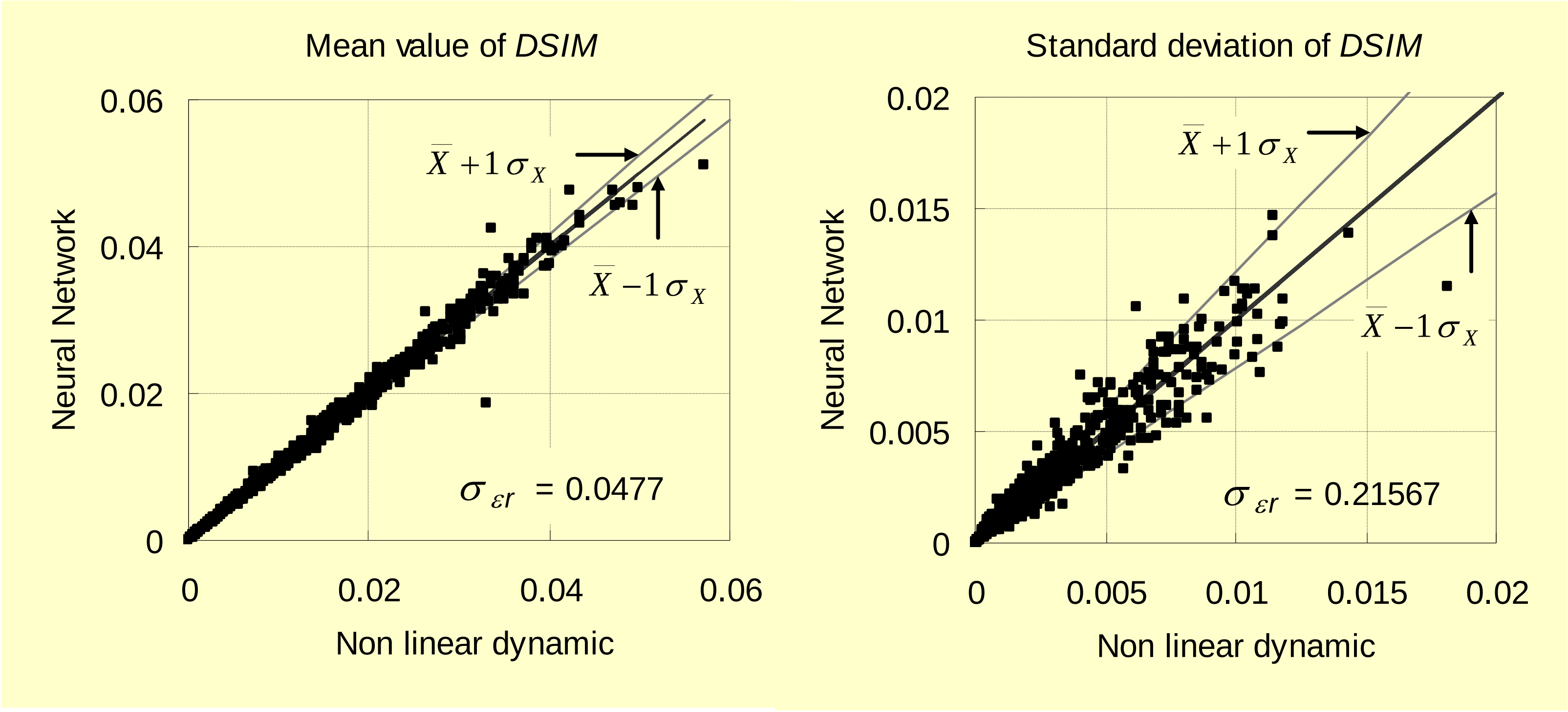
Responses can be first obtained deterministically, for a number of different combinations of the variables ***X***, choosing them appropriately over the range for each variable. Within each combination, a number N of sub-cases can be considered, each for a different earthquake record.

For each of the combinations, the mean and the standard deviation of the response parameters can then be obtained over the *N* sub-cases, (i.e., over the earthquake records):

$$\overline{R}_i = \frac{1}{N}\sum_{k=1}^{N} R_{k_i} \qquad \sigma_{R_i} = \sqrt{\frac{1}{N-1}\sum_{k=1}^{N}(R_{k_i} - \overline{R}_i)^2}$$

The means and standard deviations, assembled in two databases, are used to train two corresponding neural networks.

Neural network training results



A dispersion due to lack of fit is quantified and used when the neural network is implemented.

The mean and the standard deviation for each response can then be estimated using Normal distributions around the corresponding values calculated from the neural networks:

$$\overline{R}_i = \overline{Y_i}\,(1. + \sigma_{\varepsilon m}\, X_{N_1}) \qquad \sigma_{R_i} = \sigma_{Y_i}\,(1. + \sigma_{\varepsilon\sigma}\, X_{N_2})$$

in which $\overline{Y}_i$, σ_{Y_i} are the mean and standard deviations calculated with the corresponding neural networks and X_{N1}, X_{N2} are independent Standard Normal variables.

The mean and standard deviations were used to represent the probability distribution of each response $R(\mathbf{X})$.

■ For maximum displacements and drifts, a lognormal distribution can be used:

$$R(\mathbf{X}) = \frac{\overline{R}(\mathbf{X})}{\sqrt{1. + \left(\frac{\sigma_R(\mathbf{X})}{\overline{R}(\mathbf{X})}\right)^2}} \exp\left[R_{N_1} \sqrt{\ln\left(1. + \left(\frac{\sigma_R(\mathbf{X})}{\overline{R}(\mathbf{X})}\right)^2\right)} \right]$$

in which R_{N1} is a Standard Normal variable. (This is the same distribution used in the PEER approach, for example).

■ The variability of damage indices, being bounded between 0 and 1, was represented by Beta distributions. To calculate an individual $R(\mathbf{X})$ from the Beta distribution, a uniform random variable $0<R_{N2}<1$,was correspondingly introduced.

A Standard Montecarlo simulation can then be used, simultaneously including all limit state functions in each performance level, as in a series system.

The number of replications used varied between 10^6 to 10^7, a task significantly facilitated by the use of the neural network representations for the structural responses $R(\mathbf{X})$.

An annual probability can then be calculated taking into account the occurrence of earthquakes as a Poisson arrival process with a mean arrival rate ν:

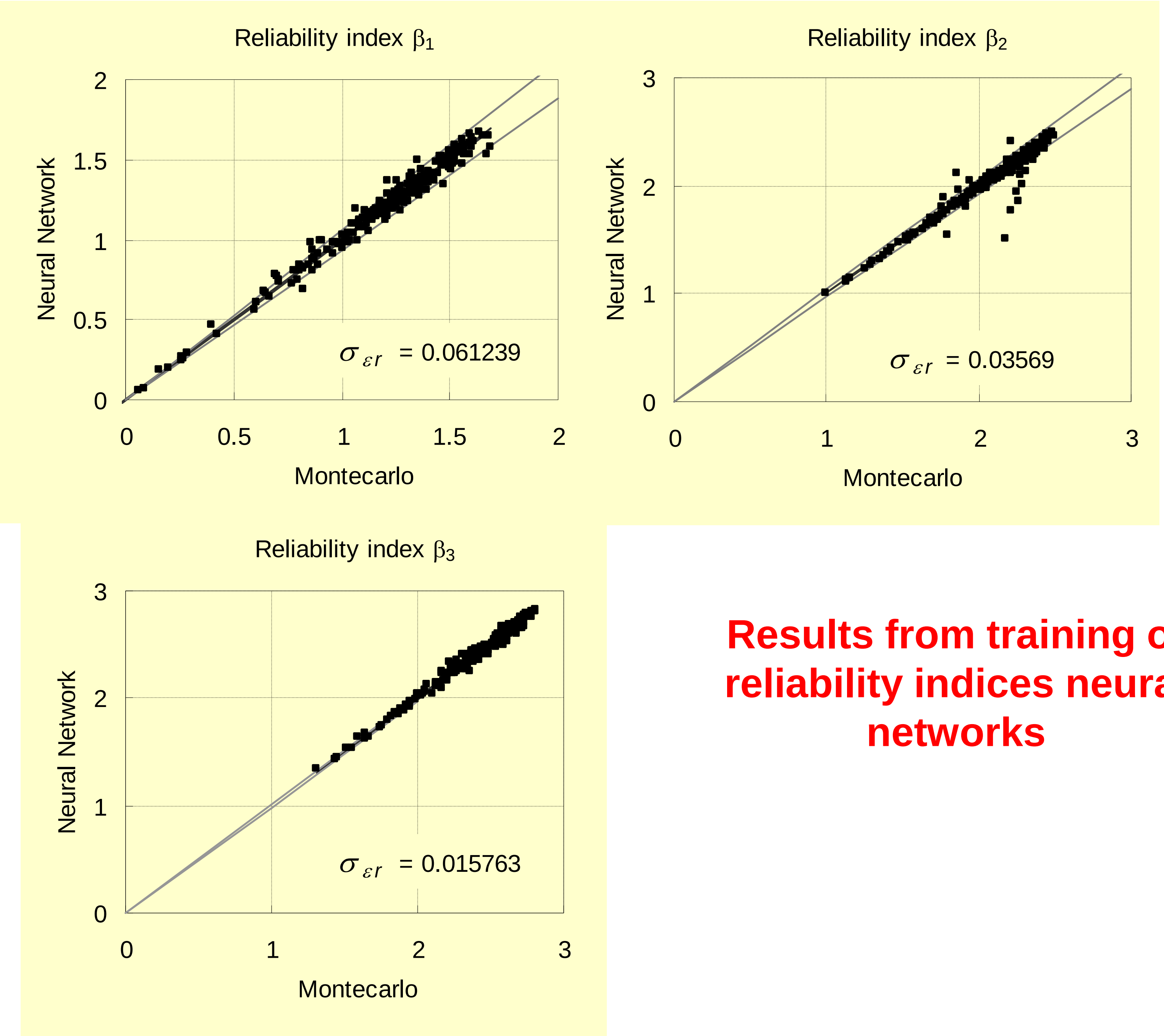
$$Pf_{annual} = 1. - \exp\left[-\nu \ Pf \ \right] \ \rightarrow \ \beta_{annual} \cong -\Phi^{-1}(Pf_{annual})$$

Neural network representation of calculated reliabilities

In order to make the optimization process more efficient, discrete databases can be obtained for the reliability indices β_1, β_2, β_3 corresponding to each of the three performance levels.

Thus, for different combinations of the design parameters, the reliability levels can be computed and then used to train three corresponding neural networks.

The advantage of this approach is that, during the optimization process, the evaluation of the achieved reliabilities can be done by executing the trained networks rather than proceeding each time with the Montecarlo simulation.



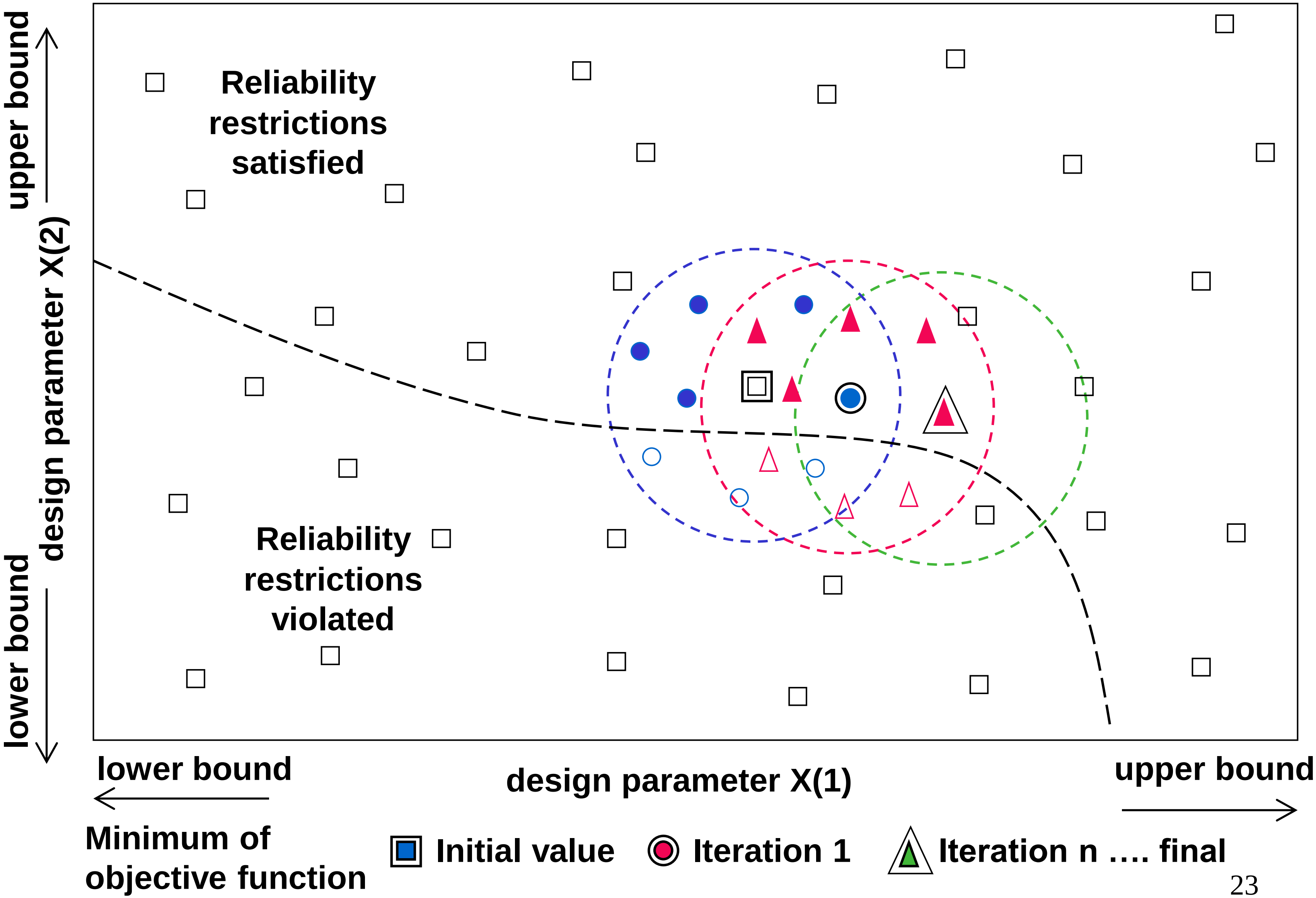
Results from training of reliability indices neural networks

OPTIMIZATION

Optimization strategy

Our problem is one of constrained optimization. The constraints are the minimum reliability levels for each performance level.

- In this work a search scheme is used which does not require the calculation of gradients (which could be prone to difficulties).


Reliability restrictions satisfied
Reliability restrictions violated
upper bound
design parameter X(2)
lower bound
lower bound
design parameter X(1)
upper bound
Minimum of objective function
Initial value
Iteration 1
Iteration n …. final

NUMERICAL RESULTS

Design parameter	Cycle 1	Cycle 2	Cycle 3
$d_1 = \overline{X}(7) = h_b\ (cm)$	60.41	59.76	61.23
$d_2 = \overline{X}(9) = h_c\ (cm)$	53.10	53.06	50.71
$d_3 = \overline{X}(10) = \rho_{sb}$	0.01226	0.01171	0.00782
$d_4 = \overline{X}(11) = \rho'_{ss}$	0.01366	0.01270	0.01310
$d_5 = \overline{X}(12) = \rho_{sc}$	0.01916	0.01782	0.02447
Initial cost ($)	10712	10436	10665
Repair cost ($)	1286	1419	1280
Total cost ($)	11998	11855	11945
β_1 ($\beta_{1T} = 1.276$)	1.4179	1.4062	1.4560
β_2 ($\beta_{2T} = 2.326$)	2.3304	2.3306	2.4050
β_3 ($\beta_{3T} = 2.697$)	2.7098	2.7006	2.7148

■ The following table shows the optimum results when using ten complete optimization cycles.

Design parameter			
$d_1 = \overline{X}(7) = h_b\ (cm)$	61.29	Initial cost ($)	$\boxed{10466}$
		Repair cost ($)	$\boxed{1256}$
$d_2 = \overline{X}(9) = h_c\ (cm)$	51.78	Total cost ($)	$\boxed{11722}$
$d_3 = \overline{X}(10) = \rho_{sb}$	0.00899	β_1 ($\beta_{1T} = 1.276$)	1.4483
$d_4 = \overline{X}(11) = \rho'_{ss}$	0.01294	β_2 ($\beta_{2T} = 2.326$)	2.3831
$d_5 = \overline{X}(12) = \rho_{sc}$	0.02383	β_3 ($\beta_{3T} = 2.697$)	2.7054

CONCLUSIONS

- Performance-based design implies an optimization process, minimizing an objective while satisfying minimum reliability levels at each performance level.

- This work has shown the implementation of neural networks for the representation of structural responses, making feasible the calculation of reliability via simulation. The reliabilities themselves can be represented by neural networks, making more efficient the optimization. A neural network is just a form of a response surface, and a different surface form could have been used. Neural networks offer a very useful tool to represent the relationship between structural responses and the intervening random variables, and between achieved reliabilities and the design parameters.

- A simple search optimization algorithm has been applied which does not require the computation of gradients (prone to problems!).

CONCLUSIONS (Cont.)

Further research needs:

1) Damage indicators and their relationship to quantities that we calculate from the dynamic analysis.

2) Relationship between the damage indicator and repair costs, including variability.

3) Further developments in nonlinear dynamic analysis, calculating rather than using a-priori hysteresis characteristics.

4) Time reliability issues related to degradation (for example, corrosion and concrete deterioration).

Thank you very much !